3. The numbers 1 to 30 inclusive are written on 30 identical slips of paper, placed in a box and thoroughly mixed. One slip of paper is chosen at random from the box. Find the probability that the number printed on the slip is:
(i) odd **(ii)** less than 7 **(iii)** divisible by 5 **(iv)** divisible by 9
(v) a two-digit number **(vi)** a perfect square **(vii)** a prime number.

4. A card is drawn at random from a normal pack of 52 playing cards.
What is the probability that the card will be:
(i) the nine of spades **(ii)** a red card **(iii)** a club
(iv) a king **(v)** a picture card **(vi)** a black picture card
(vii) an even number **(viii)** not a queen **(ix)** a joker?

5. A die is thrown 120 times. How many times would you expect the die to land on six?

6. 1,000 tickets are sold in a raffle. There is only one prize.
How many tickets does a person need to buy to have exactly 1 chance in 5 (i.e. $\frac{1}{5}$) of winning?

7. A bag contains 3 red, 3 green and 4 blue discs. A disc is selected at random from the bag. What is the probability of selecting a blue disc?
The selected disc is to be put back into the bag, plus a certain number of red discs. This causes the probability of selecting a red disc to equal $\frac{1}{2}$.
Find the number of extra red discs that were placed in the bag.

8. When a die is thrown, an odd number occurs. What is the probability that the number is prime?

9. A card is chosen at random from a set of twenty-five cards numbered from 1 to 25. What is the probability that the card chosen is a multiple of 4, given that it is greater than 15?

10. Two fair dice are thrown. Find the probability that one of the dice shows a four, given that the total on the two dice is 10.

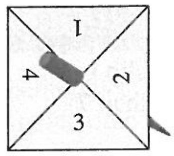
11. A box contains 20 blue counters and 30 green counters. Each counter is numbered with an even or odd number. 5 of the blue and 20 of the green counters are odd. Complete the table opposite.

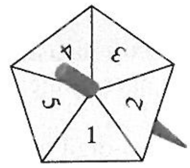
	Even	**Odd**	**Total**
Blue			20
Green			
Total			50

One of the counters is chosen at random.
What is the probability that the counter is:
(i) blue **(ii)** green **(iii)** blue and even **(iv)** green and odd?
A green counter is chosen at random.
(v) What is the probability that it is odd?
An odd-numbered counter is chosen at random.
(vi) What is the probability that it is blue?

12. There are 80 members in a club, 32 male and 48 female. 4 of the males and 8 of the females wear glasses. A club member is selected at random.
What is the probability that the club member is a:
(i) male **(ii)** female **(iii)** person wearing glasses
(iv) female not wearing glasses **(v)** male wearing glasses?
A male from the club is selected at random.
(vi) What is the probability that he wears glasses?
A member who wears glasses is selected at random.
(vii) What is the probability that it is a female?

13. A game is played with two fair spinners, as shown. Both are spun at the same time and the outcomes are added to get a score. How many scores are possible?





Calculate the probability of a score:

(i) of 4 **(ii)** of 6

(iii) greater than 6 **(iv)** less than or equal to 5.

14. A box contains 4 discs, numbered 1, 3, 3 and 4. A disc is drawn from the box and replaced. Then a second disc is drawn. A score is the sum of the two numbers drawn.

Calculate the probability that:

(i) the sum of the numbers is 4 **(ii)** the sum of the numbers is 6 or 7

(iii) the numbers drawn are the same **(iv)** the difference between the numbers is less than 2.

15. Two unbiased dice are thrown, one red and the other black.

(i) How many outcomes are possible?

(ii) If the scores are added together, calculate the probability that the sum of the scores is:

(a) less than 6 **(b)** 7 **(c)** greater than 10.

16. Two unbiased dice are thrown. Find the probability that the product of the scores is:

(i) even **(ii)** a multiple of 4 **(iii)** a multiple of 12.

17. A bag *A* contains 6 blue discs and 2 yellow discs. A bag *B* contains 4 blue discs and 2 yellow discs. A disc is drawn from bag *A* and then a disc is drawn from bag *B*.

Calculate the probability that:

(i) both discs are yellow

(ii) both discs are the same colour

(iii) the disc from bag *A* is blue and the disc from bag *B* is yellow.

18. A bag contains five discs, numbered 1, 2, 3, 4 and 5. A disc is drawn from the bag and not replaced. Then a second disc is drawn from the bag.

How many outcomes are possible?

Calculate the probability that:

(i) the sum of the outcomes is less than 5

(ii) one outcome is exactly 3 greater than the other

(iii) the difference between the outcomes is 2.

19. A box contains five cards, numbered 2, 3, 4, 4 and 5. One card is picked from the bag and not replaced. Then a second card is picked. Using a sample space diagram, or otherwise, find the probability that the numbers on the cards:

(i) are both odd **(ii)** have a sum of 6 **(iii)** have a sum of 7 or less.

Addition Rule (OR)

If A and B are two different events of the same experiment, then the probability that the two events, A or B, can happen is given by:

$$P(A \text{ or } B) = P(A) + P(B) - \underbrace{P(A \text{ and } B)}_{\text{(removes double counting)}}$$

It is often called the **or** rule. It is important to remember that $P(A$ or $B)$ means A occurs, or B occurs, or both occur. By subtracting $P(A$ and $B)$, the possibility of double counting is removed.

Example

An unbiased twenty-sided die, numbered 1 to 20, is thrown.
What is the probability of obtaining a number divisible by 4 or by 5?

Solution:

There are 20 possible outcomes.

Numbers divisible by 4 are 4, 8, 12, 16 or 20 $\quad \therefore \; P(\text{divisible by } 4) = \frac{5}{20}$

Numbers divisible by 5 are 5, 10, 15 or 20 $\quad \therefore \; P(\text{divisible by } 5) = \frac{4}{20}$

Number divisible by 4 and 5 is 20 $\quad \therefore \; P(\text{divisible by 4 and } 5) = \frac{1}{20}$

$P(\text{number divisible by 4 or } 5)$

$= P(\text{number divisible by } 4) + P(\text{number divisible by } 5) - P(\text{number divisible by 4 and } 5)$

(removes the double counting of the number 20)

$$= \frac{5}{20} + \frac{4}{20} - \frac{1}{20}$$

$$= \frac{8}{20} = \frac{2}{5}$$

The number 20 is common to both events and if the probabilities were simply added, then the number 20 would have been counted twice.

Example

A single card is drawn at random from a pack of 52. What is the probability that it is a king or a spade? What is the probability that it is not a king or spade?

Solution:

Let K represent that a king is chosen and S represent that a spade is chosen.
The pack contains 52 cards.

There are 4 kings in the pack, $\therefore \quad P(K) = \frac{4}{52}$

There are 13 spades in the pack, $\therefore \quad P(S) = \frac{13}{52}$

One card is both a king and a spade, $\therefore \quad P(K \text{ and } S) = \frac{1}{52}$

(i) $P(K \text{ or } S) = P(K) + P(S) - P(K \text{ and } S)$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

(ii) $P(\text{not } K \text{ or } S) = 1 - P(K \text{ or } S)$

$$= 1 - \frac{4}{13}$$

$$= \frac{9}{13}$$

Example

A bag contains five red, three blue and two yellow discs. The red discs are numbered 1, 2, 3, 4 and 5; the blue discs are numbered 6, 7 and 8; and the yellow discs are numbered 9 and 10. A single disc is drawn at random from the bag. What is the probability that the disc is blue or even?

Solution:

There are 10 possible outcomes.

Let B represent that a blue disc is chosen and E represent that a disc with an even number is chosen.

$P(B \text{ or } E) = P(B) + P(E) - P(B \text{ and } E)$

$$= \frac{3}{10} + \frac{5}{10} - \frac{2}{10}$$ (removes double counting)

$$= \frac{6}{10} = \frac{3}{5}$$

Exercise 5.4

1. An unbiased die is thrown.
Find the probability that the number obtained is:
(i) even **(ii)** prime **(iii)** even or prime.

2. A number is chosen at random from the whole numbers 1 to 12 inclusive.
What is the probability that it is:
(i) even **(ii)** divisible by 3 **(iii)** even or divisible by 3 **(iv)** not even or divisible by 3?

3. A number is chosen at random from the whole numbers 1 to 30 inclusive.
What is the probability that it is divisible by:
(i) 3 **(ii)** 5 **(iii)** 3 or 5 **(iv)** not 3 or 5?

4. A letter is selected at random from the word *EXERCISES*.
Find the probability that the letter is:
(i) *I* **(ii)** *S* **(iii)** a vowel **(iv)** a vowel or an *S* **(v)** not a vowel or an *S*.

5. A bag contains three blue discs, five white discs and four red discs.
A disc is chosen at random.
Find the probability that the disc chosen is:
(i) red **(ii)** blue or white **(iii)** red or white **(iv)** not red or white.

6. In a class of 20 students, 4 of the 9 girls and 3 of the 11 boys play on the school hockey team.
A student from the class is chosen at random. What is the probability that the student chosen is:
(i) on the hockey team **(ii)** a boy
(iii) a boy or on the hockey team **(iv)** a girl or not on the hockey team?

7. In lotto, there are 42 numbers, numbered from 1 to 42.
Find the probability that the first number drawn is:
(i) an even number **(ii)** a number greater than 24
(iii) an odd number or a number greater than 24 **(iv)** a number divisible by 6
(v) a number divisible by 4 **(vi)** a number divisible by 6 or 4
(vii) not a number divisible by 6 or 4.

8. A card is selected at random from a pack of 52.
Find the probability that the card is:
(i) a spade or a club **(ii)** a queen or a red card
(iii) a heart or a red picture card **(iv)** not a heart or a red picture card.

9. Two unbiased dice, one red and the other blue, are thrown together.
Calculate the probability that:
(i) the numbers are the same or the sum of the numbers is 6
(ii) the sum of the numbers is 8 or the difference between the two numbers is 2.

10. A bag contains five red discs and three blue discs. The red discs are numbered 1, 2, 2, 3 and 3, while the blue discs are numbered 4, 5 and 5. A single disc is drawn at random from the bag.
What is the probability that the disc is:
(i) red **(ii)** even **(iii)** red or even **(iv)** neither red nor even?

11. A bag contains five purple markers, four green markers and three black markers. The purple markers are numbered 1, 2, 3, 4, and 5; the green markers are numbered 6, 7, 8 and 9; while the black markers are numbered 10, 11 and 12. A single marker is drawn from the bag.
What is the probability that the marker is:
(i) odd **(ii)** black **(iii)** black or odd **(iv)** purple or even
(v) green or even **(iv)** not green or even?

Multiplication Rule (AND)

Successive Events

> The probability that two events, A and then B, both happen and in that order, is given by:
> $$P(A \text{ and } B) = P(A) \times P(B)$$
> where $P(B)$ has been worked out assuming that A has already occurred.

Order must be taken into account. Also, be very careful where the outcome at one stage does affect the outcome at the next stage. This rule also applies to more than two events.

> When the question says **and**, then multiply.

Example

A bag contains 3 red and 2 yellow discs only. When a disc is drawn from the bag, it is returned before the next draw. What is the probability that two draws will yield both discs the same colour?

Solution:

Method 1: Using a sample space diagram

Let R represent that a red disc is chosen and let Y represent that a yellow disc is chosen.

sample space diagram

second selection					
Y				•	•
Y				•	•
R	•	•	•		
R	•	•	•		
R	•	•	•		
	R	R	R	Y	Y

first selection

There are 25 possible outcomes
(5 for the first draw and 5 for the second draw).
The dots indicate where the colours are the same, successful outcome, either two reds or two yellows. There are 13 dots.

$$P(\text{both discs the same colour}) = \frac{13}{25}$$

Method 2: Picking one at a time with replacement

Let R_1 represent that a red disc is chosen first, Y_2 represent that a yellow disc is chosen second, and so on.

P(both discs the same colour)

$$= P(R_1 \text{ and } R_2) \quad \text{or} \quad P(Y_1 \text{ and } Y_2)$$

$$= P(R_1) \times P(R_2) \quad + \quad P(Y_1) \times P(Y_2)$$

$$= \frac{3}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{2}{5}$$

$$= \frac{9}{25} + \frac{4}{25}$$

$$= \frac{13}{25}$$

> red and then a red
> or
> yellow and then a yellow

Method 3: Using arrangements

$$\begin{array}{lccccccc} & \text{red and red} & & \text{or} & \text{yellow and yellow} & & \\ \text{Number of desirable outcomes} = & 3 \times 3 & & + & 2 \times 2 & & \text{(discs are replaced)} \\ & = 9 + 4 = 13 & & & & & \end{array}$$

Number of possible outcomes $= 5 \times 5 = 25$

$\therefore$ $P(\text{both discs the same colour}) = \dfrac{13}{25}$

Note: A combinations approach will not work in this example, as the disc was returned before the next disc was picked.

Example

A bag contains four red discs and five blue discs. Three discs are selected at random. Find the probability that two are red and one is blue.

Solution:

(i) Let R_1 represent that a red disc is chosen first, B_2 represent that a blue disc is chosen second, and so on. Three discs selected at random is equivalent to selecting one disc after another **without** replacement.

Method 1: Picking one at a time without replacement

Two reds and one blue can occur in three ways:

$$\begin{array}{lccccc} & R_1 \text{ and } R_2 \text{ and } B_3 & \text{or} & R_1 \text{ and } B_2 \text{ and } R_3 & \text{or} & B_1 \text{ and } R_2 \text{ and } R_3 \\ & P(\text{two red and blue}) & & & & \\ & = P(R_1 \text{ and } R_2 \text{ and } B_3) & \text{or} & P(R_1 \text{ and } B_2 \text{ and } R_3) & \text{or} & P(B_1 \text{ and } R_2 \text{ and } R_3) \\ = & \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} & + & \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} & + & \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \\ = & \frac{5}{42} & + & \frac{5}{42} & + & \frac{5}{42} \\ = & \frac{15}{42} = \frac{5}{14} & & & & \end{array}$$

Method 2: Using combinations

There are 9 discs and we want to choose 3.

The total number of selections of 3 from $9 = \binom{9}{3} = 84$.

The number of ways of selecting two red and one blue $= \binom{4}{2} \times \binom{5}{1} = 6 \times 5 = 30$

$\therefore$ $P(\text{two red and one blue}) = \dfrac{30}{84} = \dfrac{5}{14}$

Method 3: Using arrangements

$$\begin{array}{lccccccc} \text{Number of desirable outcomes} = & R\,R\,B & \text{or} & R\,B\,R & \text{or} & B\,R\,R & & \\ & = 4 \times 3 \times 5 & + & 4 \times 5 \times 3 & + & 5 \times 4 \times 3 & & \\ & = 60 & + & 60 & + & 60 & = & 180 \end{array}$$

Number of possible outcomes $= 9 \times 8 \times 7 = 504$

$\therefore$ $P(\text{two red and one blue}) = \dfrac{180}{504} = \dfrac{5}{14}$

Example

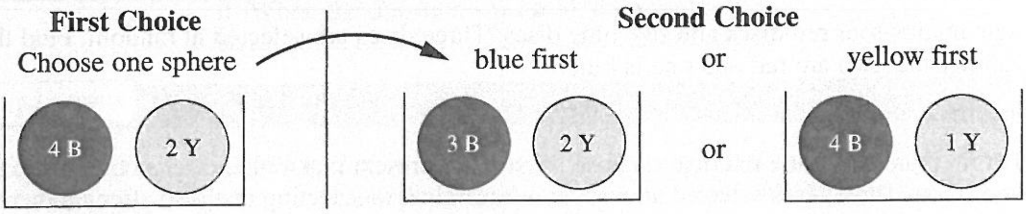
A box contains four blue spheres and two yellow spheres. One sphere is removed at random and not replaced. Then a second sphere is removed at random. Find the probability that one sphere is blue and the other is yellow.

Solution:

Let B_1 represent that a blue sphere is chosen first, Y_2 represent that a yellow is chosen second, and so on.

Diagram of the situation

There are two possible sample spaces for the second choice. It depends on whether a blue sphere is chosen first or a yellow sphere is chosen first.



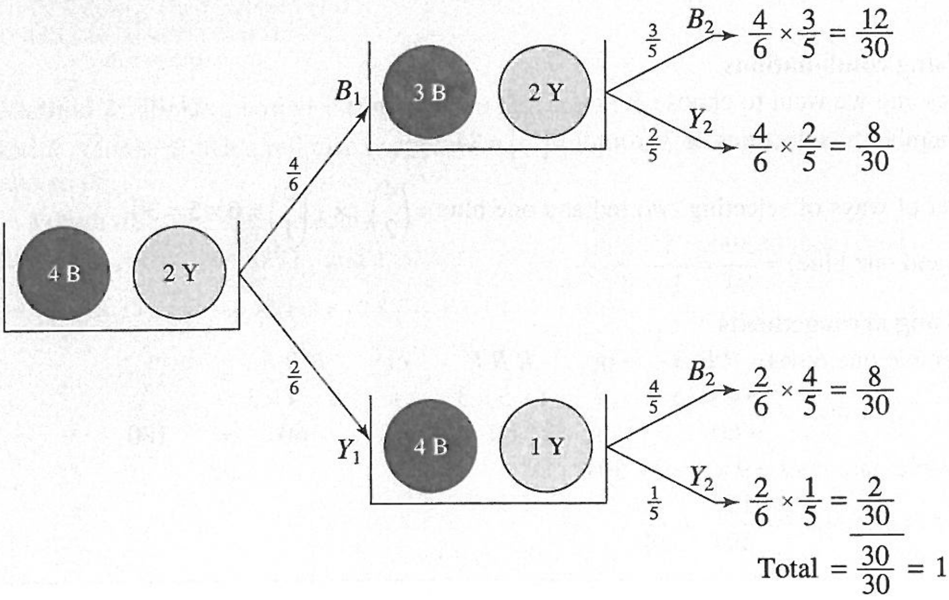
Method 1: Picking one at a time without replacement

P(one sphere is blue and the other is yellow)

$$\begin{aligned}
&= P(B_1 \text{ and } Y_2) \quad \text{or} \quad P(Y_1 \text{ and } B_2)\\
&= P(B_1) \times P(Y_2) \quad + \quad P(Y_1) \times P(B_2)\\
&= \frac{4}{6} \times \frac{2}{5} \quad + \quad \frac{2}{6} \times \frac{4}{5}\\
&= \frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}
\end{aligned}$$

blue first and then a yellow second or yellow first and then a blue second

Method 2: Using a probability tree diagram



Multiply the probabilities along the branches to get the end results.
If more than one end result is required, add these results together.

$P(\text{one sphere is blue and the other is yellow}) = \frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}$

Method 3: Using combinations

As the first sphere is not replaced, combinations can be used.
There are 6 spheres and we want to choose 2.

The total number of selections of 2 from $6 = \binom{6}{2} = 15$.

The number of ways of selecting one blue and one yellow $= \binom{4}{1} \times \binom{2}{1} = 4 \times 2 = 8$.

$\therefore \quad P(\text{one sphere is blue and the other is yellow}) = \frac{8}{15}$

Method 4: Using arrangements

$$\begin{array}{lccc} & B \text{ and } Y & \text{or} & Y \text{ and } B \\ \text{Number of desirable outcomes} = & 4 \times 2 & + & 2 \times 4 \\ = & 8 + 8 = 16 & & \end{array}$$

Number of possible outcomes $= 6 \times 5 = 30$ (sphere not replaced)

$\therefore \quad P(\text{one sphere is blue and the other is yellow}) = \frac{16}{30} = \frac{8}{15}$

Example

A bag contains 16 marbles, 6 of which are white and the remainder black.
Three marbles are removed at random, one at a time, without replacement.
Find the probability that:

(i) all are black **(ii)** at least one is white.

Solution:

Let B_1 represent that a black marble is chosen first, B_2 that a black marble is chosen second, and B_3 that a black marble is chosen third.
There are 20 marbles, 6 white, 10 black.

(i) $P(\text{all are black}) = P(B_1) \times P(B_2) \times P(B_3)$

$$= \frac{10}{16} \times \frac{9}{15} \times \frac{8}{14}$$

$$= \frac{3}{14}$$

$P(B_1) = \frac{10}{16}$

$P(B_2) = \frac{9}{15}$ $\left(\begin{array}{l}\text{1 black removed,} \\ \text{9 black and 6 white left}\end{array}\right)$

$P(B_3) = \frac{8}{14}$ $\left(\begin{array}{l}\text{2 blacks removed,} \\ \text{8 black and 6 white left}\end{array}\right)$

(ii) In every other case there is **at least** one white.

$P(\text{at least one white}) = 1 - P(\text{none is white})$

$$= 1 - P(\text{all are black})$$

$$= 1 - \frac{3}{14}$$

$$= \frac{11}{14}$$

Note: A combinations or arrangement approach would also work.

Example

Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs, are drawn at random (without replacement) from the box.

(i) What is the probability that the disc with the number 7 is drawn?

(ii) What is the probability that the three numbers on the discs drawn are odd?

(iii) What is the probability that the product of the three numbers on the discs drawn is even?

(iv) What is the probability that the smallest number on the discs drawn is 4?

Solution:

10 discs, numbered from 1 to 10 inclusive.

Method 1: Picking one at a time without replacement

(i) P(disc with the number 7)

$= P(7, \text{not } 7, \text{not } 7) \quad \text{or} \quad P(\text{not } 7, 7, \text{not } 7) \quad \text{or} \quad P(\text{not } 7, \text{not } 7, 7)$

$$= \frac{1}{10} \times \frac{9}{9} \times \frac{8}{8} + \frac{9}{10} \times \frac{1}{9} \times \frac{8}{8} + \frac{9}{10} \times \frac{8}{9} \times \frac{1}{8}$$

$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

(ii) P(three numbers on the discs are odd)

$= P(\text{1st odd}) \times P(\text{2nd odd}) \times P(\text{3rd odd})$

$$= \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720} = \frac{1}{12}$$

(iii) Product means the result of multiplying.

If at least one number is even, then the product of the three numbers will be even.

P(product of the three numbers on the discs is even)

$= P(\text{at least one even number})$

$= 1 - P(\text{three numbers on the discs are odd})$

$$= 1 - \frac{1}{12} = \frac{11}{12}$$

Alternatively, let E_1 represent that an even number is picked first, O_2 represent that an odd number is picked second, and so on.

P(product of the three numbers on the discs is even)

$$= P(E_1, E_2, E_3) + P(E_1, E_2, O_3) + P(E_1, O_2, E_3) + P(O_1, E_2, E_3) + P(E_1, O_2, O_3) + P(O_1, E_2, O_3) + P(O_1, O_2, E_3)$$

$$= P(E_1, E_2, E_3) + 3P(E_1, E_2, O_3) + 3P(E_1, O_2, O_3)$$

$$= \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} + 3 \times \frac{5}{10} \times \frac{4}{9} \times \frac{5}{8} + 3 \times \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8}$$

$$= \frac{60}{720} + \frac{300}{720} + \frac{300}{720} = \frac{660}{720} = \frac{11}{12}$$

(iv) For the smallest number to be 4, we need to choose 4 and any two from 5, 6, 7, 8, 9, 10.

P(smallest number is 4)

$= P(4, \text{not } 4, \text{not } 4) + P(\text{not } 4, 4, \text{not } 4) + P(\text{not } 4, \text{not } 4, 4)$

$$= \frac{1}{10} \times \frac{6}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{1}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{5}{9} \times \frac{1}{8}$$

$$= \frac{30}{720} + \frac{30}{720} + \frac{30}{720} = \frac{90}{720} = \frac{1}{8}$$

Method 2: Using combinations

As the discs are **not** replaced, we can use combinations.

In each case the number of possible outcomes $= \binom{10}{3} = 120.$

(i) One 7 and any two others:

Number of favourable outcomes $= \binom{1}{1} \times \binom{9}{2} = 1 \times 36 = 36$

$P(\text{disc with the number } 7) = \frac{36}{120} = \frac{3}{10}$

(ii) Three numbers on the discs are odd:

Number of favourable outcomes $= \binom{5}{3} = 10$

$P(\text{three numbers on the discs are odd}) = \frac{10}{120} = \frac{1}{12}$

(iii) Product of the three numbers on the discs is even:

The product of the three numbers is either even or odd.

$$\therefore \quad \begin{pmatrix}\text{number of possible}\\ \text{outcomes}\end{pmatrix} = \begin{pmatrix}\text{number of outcomes}\\ \text{where the product is even}\end{pmatrix} + \begin{pmatrix}\text{number of outcomes}\\ \text{where the product is odd}\end{pmatrix}$$

$$\begin{pmatrix}\text{number of outcomes where}\\ \text{the product is even}\end{pmatrix} = \begin{pmatrix}\text{number of possible}\\ \text{outcomes}\end{pmatrix} - \begin{pmatrix}\text{number of outcomes where}\\ \text{the product is odd}\end{pmatrix}$$

$$= \binom{10}{3} - \binom{5}{3}$$

$$= 120 - 10 = 110$$

$P(\text{product of the three numbers on the disc is even}) = \frac{110}{120} = \frac{11}{12}$

(iv) The smallest number on the discs is 4:

choose 4 and any two from 5, 6, 7, 8, 9, 10.

Number of favourable outcomes $= \binom{1}{1} \times \binom{6}{2} = 1 \times 15 = 15$

$P(\text{smallest number is } 4) = \frac{15}{120} = \frac{1}{8}$

Example

In a particular week (Monday to Sunday inclusive), three students, A, B and C, celebrate their birthdays. Assume that the birthdays are equally likely to fall on any day of the week and that the birthdays are independent of each other. What is the probability that:

(i) A has a birthday on Tuesday
(ii) B and C have their birthday on a Wednesday
(iii) B and C have their birthday on the same day
(iv) none of them has a birthday on Sunday
(v) at least two of them share the same birthday?

Solution:

P(any person has a birthday on a particular day of the week) $= \dfrac{1}{7}$

P(any person does **not** have a birthday on a particular day of the week) $= \dfrac{6}{7}$

(i) $P(A$ has a birthday on Tuesday$) = \dfrac{1}{7}$

(ii) $P(B$ and C have their birthday on Wednesday$)$

$= P(B$ has a birthday on Wednesday$) \times P(C$ has a birthday on Wednesday$)$

$$= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$$

(iii) $P(B$ and C have their birthday on the same day$)$

Method 1:

$P(B$ and C have their birthday on the same day$)$

$= P\left(\begin{array}{l}\text{both born on Monday or Tuesday or Wednesday or Thursday or Friday} \\ \text{or Saturday or Sunday}\end{array}\right)$

$$= \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7} + \frac{1}{7} \times \frac{1}{7}$$

$$= \frac{1}{49} + \frac{1}{49} + \frac{1}{49} + \frac{1}{49} + \frac{1}{49} + \frac{1}{49} + \frac{1}{49} = \frac{7}{49} = \frac{1}{7}$$

Method 2:

$P(B$ and C have their birthday on the same day$)$

$= P(B$ has a birthday on some day of the week$) \times P(C$ has a birthday on the same day$)$

$$= \frac{7}{7} \times \frac{1}{7} = \frac{1}{7}$$

(iv) P(none of them has a birthday on Sunday)

$= P\left(\begin{array}{l}A \text{ does not have a} \\ \text{birthday on Sunday}\end{array} \text{ and } \begin{array}{l}B \text{ does not have a} \\ \text{birthday on Sunday}\end{array} \text{ and } \begin{array}{l}C \text{ does not have a} \\ \text{birthday on Sunday}\end{array}\right)$

$$= \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} = \frac{216}{343}$$

(v) P(at least two of them share the same birthday)

$P(A$ has a birthday on some day of the week$) = \dfrac{7}{7} = 1$

$P(B$ has a birthday on a different day from $A) = \dfrac{6}{7}$

$P(C$ has a birthday on a different day from A and $B) = \dfrac{5}{7}$

$\therefore$ P(all three have birthdays on different days) $= 1 \times \dfrac{6}{7} \times \dfrac{5}{7} = \dfrac{30}{49}$

$\therefore$ P(at least two of them share the same birthday)

$= 1 - P$(all three have birthdays on different days)

$= 1 - \dfrac{30}{49} = \dfrac{19}{49}$

Example

A box contains seven silver coins, four gold coins and x copper coins.
Two coins are picked at random, and without replacement, from the box.
Write down an expression in x for the probability that the two coins are both copper.

If it is known that the probability of picking two copper coins is $\dfrac{3}{14}$, how many copper coins are in the box?

Solution:

7 silver coins, 4 gold coins and x copper coins.
Thus, the total number of coins $= x + 11$

$P(\text{copper coin first}) = \dfrac{x}{x+11}$

First draw

x copper coins
and
$x + 11$ coins in total

$P(\text{copper coin second}) = \dfrac{x-1}{x+10}$

Second draw

Assuming a copper coin is drawn first:
$x - 1$ copper coins left
and
$x + 10$ coins in total

Given: $P(\text{copper and then a copper coin}) = \dfrac{3}{14}$

$$P(\text{copper first}) \times P(\text{copper second}) = \frac{3}{14}$$

$$\frac{x}{x+11} \times \frac{x-1}{x+10} = \frac{3}{14}$$

$$\frac{x^2 - x}{x^2 + 21x + 110} = \frac{3}{14}$$

$$14x^2 - 14x = 3x^2 + 63x + 330 \qquad \text{(multiply both sides by 14 and } x^2 + 21x + 110\text{)}$$
$$11x^2 - 77x - 330 = 0$$
$$x^2 - 7x - 30 = 0$$
$$(x - 10)(x + 3) = 0$$
$$x - 10 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 10 \quad \text{or} \quad x = -3$$

Thus, $x = 10$ (reject $x = -3$)

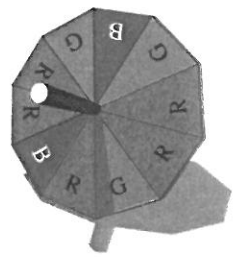
i.e., the number of copper coins is 10.

Exercise 5.5

1. Two unbiased dice are thrown. What is the probability of getting two 4s?

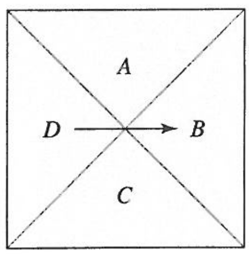
2. A fair coin is tossed and an unbiased die is thrown.
   Find the probability of:
   (i) a head and a 4
   (ii) a tail and an odd number
   (iii) a tail and a number greater than 2
   (iv) a head and a number divisible by 3.

3. A spinner used in a game has 10 sections, of which 5 are coloured red, 3 green and 2 blue.
   The spinner is spun twice. What is the probability of obtaining:
   (i) two reds
   (ii) two blues
   (iii) a red and a blue?



4. A bag contains ten marbles: five red, three blue and two yellow.
   Three marbles are drawn, one after another, without replacement. Find the probability that the first is red, the second is blue and the third is not red.

5. A game consists of spinning an unbiased arrow on a square board and throwing an unbiased die.
   The board contains the letters A, B, C and D. The board is so designed that when the arrow stops spinning it can point at only one letter, and it is equally likely to point at A, B, C or D.
   List all possible outcomes of the game, that is, of spinning the arrow and throwing the die. Find the probability that in any one game the outcome will be:
   (i) an A and a 6
   (ii) a B and an even number
   (iii) an A and an even number or a B and an odd number
   (iv) a C and a number $\geqslant 4$ or a D and a number $\leqslant 2$.



6. An unbiased die has two faces lettered A and four faces lettered B. Two boxes are labelled A and B. Box A contains 6 red and 3 white marbles. Box B contains 4 red and 5 white marbles. The die is rolled and two marbles are drawn at random, without replacement, from the box labelled with the letter uppermost on the die. Find the probability that:
(i) two white marbles are drawn from box A
(ii) both marbles are red.

7. Three coins, a 20c, a 10c and 5c, are tossed. Find the probability of getting:
(i) 3 tails **(ii)** a head and two tails.

8. Twelve blood samples are tested in a laboratory. Of these it is found that five samples are of type A, four of type B and the remaining three are of type O.
Two blood samples are selected at random from the twelve.
What is the probability that:
(i) the two samples are of type A
(ii) one sample is of type B and the other sample is of type O
(iii) the two samples are of the same blood type?

9. A bag contains 8 black discs and 4 white discs. Two discs are picked at random, one after the other, without replacement. Find the probability of drawing discs of different colours if:
(i) the first disc is replaced **(ii)** the first disc is not replaced.

10. A bag contains 5 red, 4 blue and 3 yellow balls. Three balls are removed at random, without replacement. Find the probability that the three balls are of:
(i) the same colour **(ii)** different colours.

11. A bag contains 6 red marbles and 4 white marbles. A marble is chosen at random from the bag, its colour is noted and it is returned to the bag. Then a second marble is drawn and replaced, then a third marble is drawn . Find the probability that:
(i) all three marbles drawn are the same colour
(ii) the first and last marbles are the same colour, but the middle one is different.

12. A bag contains 5 red discs and 4 blue discs. A disc is drawn at random and replaced. Then a second disc is drawn at random. What is the probability that:
(i) both discs are blue **(ii)** the second disc is blue
(iii) the same disc is drawn each time?

13. A bag contains ten blue and ten yellow spheres. Three spheres are removed at random (without replacement). Find the probability that:
(i) all are blue **(ii)** at least one is yellow.

14. Four students work separately on a mathematical problem. The probabilities that the four students have of solving the problem are as follows:

$$\frac{3}{4}, \frac{1}{2}, \frac{4}{7}, \frac{2}{3}.$$

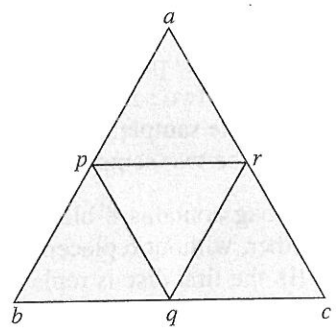
Show that the probability that the problem will be solved by at least one of the four students is $\frac{55}{56}$.

15. Three countries work separately on finding a cure for the common cold. The probabilities that the three countries have of finding a cure are as follows: $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$.
Find the probability that a cure will be found by at least one country.

16. There are three sets of traffic lights which a woman has to drive through each day on her way to work. The probability that the woman will have to stop at any of these lights is $\frac{1}{3}$. Calculate, for a particular morning, the probability that the woman will have to stop at:
(i) none of the three sets of lights **(ii)** at least one of the three sets of lights.

17. A box contains one black, one red, one yellow and three purple balls. Three balls are drawn at random from the box (without replacement).
Find the probability that:
(i) three purple balls are drawn
(ii) only two purple balls are drawn
(iii) at least two purple balls are drawn.

18. abc is a triangle. p, q and r are the midpoints of the sides, as shown in the diagram.
(i) A speck of dust falls at random onto, or inside, triangle abc. Find the probability that the speck falls within the smaller triangle pqr.
(ii) If three specks of dust fall at random onto triangle abc, find the probability that at least one falls within the smaller triangle pqr.



19. In each round of a game, a competitor can score points of 0, 1 or 2 only. Copy and complete the table which shows the points and two of the respective probabilities of these points being scored in a single round.

Points	0	1	2
Probability	$\frac{1}{2}$	$\frac{2}{5}$	

After two rounds of the game, calculate the probability that a competitor has:
(i) no points **(ii)** 3 points **(iii)** an odd number of points.

20. A man has 6 keys, which look very similar, one of which will open his hall door. On a dark night he chooses from the 6 keys, at random, without replacement, until his front door key is found. Find the probability that he will be able to open his hall door after trying:
(i) 2 keys **(ii)** 4 keys.

21. There were 9 white and 3 black marbles in a bag. In a second bag there were 7 white and 2 black marbles. If a bag and then a marble were selected at random, what was the probability of a black marble?

22. Three cards are drawn, at random and with replacement, from a pack of 52 playing cards.
Find the probability that:
(i) the three cards are red
(ii) two are red and one card is a club
(iii) the three cards are all of the same suit.

23. A bag contains nine counters, numbered 1 to 9 inclusive. Four are drawn at random without replacement. Find the probability that the four counters have digits which are:
(i) all odd **(ii)** two odd and two even.

24. There are six balls in a bag, numbered as follows: 2, 2, 3, 3, 5, 6.
Two of the balls are selected simultaneously from the bag, at random.
What is the probability that they will total 8 or more?

25. The letters of the word *TABLE* are arranged at random in order. What is the probability that the two vowels are not side by side?

26. A team of 5 people is to be chosen from 5 men and 4 women.
(i) Calculate the number of ways this can be done.
(ii) If one of the men is the husband of one of the women, and the team is selected at random, calculate the probability that the married couple will be chosen.

27. A class consists of 4 boys and 6 girls. It is proposed to use a process of random selection to pick 3 representatives for a school council.
(i) In how many ways can this be done?
(ii) How many of the selections consist of all boys or all girls?
(iii) What is the probability that the selection may be all boys or all girls?

28. A committee of two people is chosen at random from 4 men and 5 women.
(i) In how many ways can this be done?
(ii) What is the probability that there will be one man and one woman or two women on the committee?

29. A group consists of 5 boys and 4 girls. If three of the group are picked at random, what is the probability that more girls than boys are picked?

30. Eight discs, each marked with a different whole number from 1 to 8, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.
(i) What is the probability that the disc with the number 6 is drawn?
(ii) What is the probability that the three numbers on the discs drawn are odd?
(iii) What is the probability that the product of the three numbers on the discs drawn is even?
(iv) What is the probability that the smallest number on the discs drawn is 3?

31. Bag A contains 4 blue and 5 yellow counters. Bag B contains 3 blue and 6 yellow counters. A counter is picked at random from bag A and placed in bag B. A counter is then picked at random from bag B. Find the probability that the counter picked from bag B is yellow.

32. A box contains 6 white discs and 4 black discs. A disc is selected at random from the box and replaced by a disc of the other colour. A second disc is then randomly selected from the box.
Determine the probability that:
(i) the first disc selected is black and the second disc selected is white
(ii) both discs selected are white.
Assume that spare white and black discs are available.

33. Aideen and Brendan celebrate their birthdays in a particular week (Monday to Sunday inclusive). Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that:
(i) Aideen was born on Tuesday
(ii) Brendan was not born on Sunday
(iii) both were born on Friday
(iv) one was born on Thursday and the other was born on Saturday
(v) both were born on the same day
(vi) Aideen and Brendan were born on different days?

34. In a particular week (Monday to Sunday inclusive), three students celebrate their birthdays. Assume that the birthdays are equally likely to fall on any day of the week and that the birthdays are independent of each other.

What is the probability that of these three students:

(i) all were born on Thursday
(ii) all were born on the same day of the week
(iii) all were born on different days of the week
(iv) at least two of them share the same birthday
(v) one was born on Monday and the others were not born on Monday?

35. Three students were chosen at random. Assuming that the probability of being born in any given month is equiprobable, find the probability that:

(i) all three have birthdays on July
(ii) all three have birthdays in the same month
(iii) no two have their birthdays in the same month
(iv) at least two have birthdays in the same month.

36. A bag contains 10 yellow, 6 green and x blue discs. One disc is drawn at random from the bag and **not** replaced.

(i) Write down an expression in x for the probability that the disc is green.
(ii) If the probability of a green disc is $\frac{3}{10}$, find the value of x.
(iii) Another disc is then drawn from the bag.
Find the probability that both discs are of the same colour.

37. A drawer contains 6 red and x blue pens. One pen is drawn at random and **not** replaced. Another pen is then drawn at random.

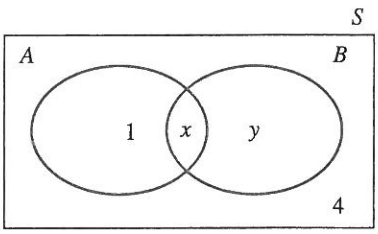
(i) Write down an expression in x for the probability that the two pens are blue.
(ii) If the probability that both are blue is $\frac{1}{12}$, find the value of x.
(iii) Find the probability that the two pens are of different colours.

38. A bag contains five yellow marbles, four blue marbles and x red marbles. Two marbles are picked at random, and without replacement, from the box.

(i) Write down an expression in x for the probability that the two marbles are red.
(ii) It is known that the probability of picking two red marbles is $\frac{1}{7}$.

(a) Find the value of x. **(b)** How many marbles are in the bag?
(c) What is the probability that the marbles are of different colours?

39. The Venn diagram shows the number of elements in each subset of the set S.
An element is picked at random from S. Write an expression in terms of x and y for the probability that the element came from: **(i)** set A **(ii)** set B.
If $P(A) = \frac{3}{10}$ and $P(B) = \frac{1}{2}$, find the value of x and the value of y.



40. A die is weighted so that a score of 5 is three times as likely to appear as a score of 4. A score of 4 is twice as likely as a score of 6, and scores of 1, 2, 3 and 6 are equally likely. On one throw of the die, find the probability of getting an odd number.